Abstract—Swarms of autonomous underwater vehicles (AUVs) forming mobile underwater networks often operate in moving currents, which introduce severe turbulence that interferes with coordinated and stealthy navigation of the fleets. Therefore, individual AUV must adjust its heading whenever needed to ensure it can reach a pre-determined destination. To achieve accurate navigation, AUVs must maintain precise knowledge of their locations. This paper develops the “Suave” (Swarm underwater autonomous vehicle localization) algorithm to localize swarms of AUVs operating in rough waters. The purpose of Suave is to ensure that all AUVs arrive at their destinations by preserving localization throughout the entire mission. Suave lowers the probability that an AUV swarm is detected by reducing the number of occasions that vehicles must surface to obtain accurate location information from external sources such as satellites. The Suave algorithm also achieves better energy conservation through improved control of localization reference messages. Simulations show Suave significantly improves localization accuracy, lowers energy consumption, and the probability of swarm detection.

I. INTRODUCTION

In recent years, advances in autonomous underwater vehicle (AUVs) have facilitated a wide variety of exciting scientific applications, including automated ship hull inspection [1], [2], autonomous underwater cave exploration [3], archaeological site mapping [4], [5], and surveys of underwater environments [6]–[8]. AUVs are very promising because they eliminate the need to control vehicles remotely. However, many research challenges remain unsolved including research related to underwater sensor networks [9]–[11]. Localization is one of the main unaddressed problems. The critical importance of localization arises from its role in fundamental operations like motion planning and map building, which are prerequisite capabilities for virtually all underwater surveying missions because data collected will only be useful when the localization information is accurate. Localization is also essential for an AUV to maintain accurate knowledge of its position, orientation, and velocity, which are needed to safeguard the vehicle from collisions with other underwater objects and the seafloor.

The unavailability of global positioning system (GPS) technology in the underwater environment further complicates the task of localizing AUVs. Many applications rely on an Inertial Measurement Unit (IMU) for navigation. However, high performance IMUs are prohibitively expensive and can only provide an estimate of a vehicle’s inertial movement, indicating that such technology cannot provide knowledge of absolute position. Thus, if the original position is unknown or inaccurate, IMUs will be of limited use for accurate navigation. Moreover, the IMU technology suffers from error accumulation. This accumulation occurs as a vehicle’s guidance system continuously updates its location according to the most recent IMU reading. Errors in successive IMU measurements produce drifts that can lead to ever-increasing disagreement between the actual and estimated location, degrading localization efforts. The Doppler Velocity Log (DVL) method [12] to estimate a vehicle’s velocity vector also suffers from similar limitations, inhibiting accurate localization. It turns out that acoustic communication [13] is suitable for absolute position estimation in underwater networks because it enables accurate ranging, and does not suffer from error accumulation inherent in path based approaches when combined with surfacing and re-localization.

This paper tackles the localization problem for an AUV swarm traveling in an underwater environment to facilitate the execution of a set of coordinated navigational maneuvers. The entire procedure consists of swarm deployment, navigation, and occasional surfacing to enable localization. We propose the Suave algorithm to improve the accuracy of AUV localization, which consists of two key phases. The first phase selects the most suitable reference AUVs for surfacing to re-localize from satellite information so that they can assist the remaining AUVs to localize after resubmerging. In the second phase, the transmission time of the reference AUVs is coordinated to ensure that the remaining AUVs receive and decode reference messages virtually the same time so that the impact of mobility during transmission and propagation time can be reduced. Suave also employs the interactive multiple model (IMM), an advanced tracking scheme, to improve the accuracy of AUV localization during period of submergence.

The remainder of this paper is organized as follows. Section II presents design challenges. Section III describes the steps of the Suave algorithm. The key components of reference AUV selection and transmission time adjustment are discussed in Section IV, and Section V shows the error analysis. Simulation results are provided in Section VI. Related work is reviewed in Section VII, followed by conclusions and future work in Section VIII.

II. DESIGN CHALLENGES

Due to the unique characteristics of the underwater acoustic channels and environment, underwater vehicle networks (UVNs) differ dramatically from terrestrial mobile radio networks. For example, low bandwidth, high latency, vehicle mobility, and energy constraints all impact
the localization process and must therefore be considered when designing an AUV localization algorithm.

A. Low bandwidth and high latency

The high attenuation of radio signals in water leaves acoustic waves the most viable communication media. However, acoustic bandwidth underwater is quite limited due to absorption. Most acoustic systems operate below 30kHz. Furthermore, the bandwidth available to underwater acoustic channels heavily depend on both the desired transmission range and frequency. According to [14], virtually no theoretical or commercial system can exceed 40km × kbps as the maximum attainable product of range × rate. Moreover, propagation latency is significantly higher because of low propagation speeds ($1.5 \times 10^3$ m/second), compared to radio communication ($3 \times 10^8$ m/second), which is five orders of magnitude lower. Thus, for a fixed-sized localization reference packet, the delays associated with transmission and propagation are noticeably longer in UVNs than ground-based mobile radio network. For these reasons, the negative impact of vehicle movement during signal transmission and propagation must be considered to avoid introducing inaccuracies into localization calculations.

B. Mobility and uncertainty

Objects residing in an underwater environment commonly exhibit passive mobility from water currents or active mobility exhibited by autonomous platforms. Empirical observations suggest that underwater objects may move at speeds of two to three knots (or three to six kilometers per hour) in a typical underwater setting. This mobility and uncertainty complicate the localization process because vehicle motion coupled with long delays can introduce significant error into range estimates.

C. Energy constraints

Underwater vehicles are often battery-powered and their relative inaccessibility renders it difficult to replace this limited power supply when exhausted. Thus, the lifetime of an AUV is severely restricted by its local power source. For this reason, the energy consumed during the localization process must also be carefully controlled. Computation and communication are the two major energy costs associated with localization. Computation consists of calculations performed on the vehicle platform chip to achieve localization, while communication relates to the energy consumed during the transmission and reception of localization reference packets. The amplifier and transducer of an acoustic modem typically needs tens of watts to reliably transmit signals to another AUV within a reasonable distance. However, the power consumption of most processing chips is on the order of a single watt or less. This means that the energy consumed by acoustic communication in UVNs is much higher than computation. Thus, the number of localization reference packets exchanged must be kept low to conserve energy.

III. DESCRIPTION OF SUAVE

A. Background and Assumptions

Fig. 1 shows an AUVs swarm, where each vehicle may be assigned a different task during travel and possesses the destination. Every AUV is equipped with a control theoretic navigation system and takes position information as input, so that its heading can be revised to stay on course. During the mission, the AUV’s velocity is the combination of the velocities of its internal propulsion system and the water current, which is unknown. Therefore, to successfully navigate to a destination, an AUV must re-localize whenever needed by providing accurate location estimates as input to its navigation system.

In Suave, All AUVs are assumed to be well synchronized, so that range can be accurately estimated with time of arrival. It is also assumed that each AUV is outfitted with a depth sensor, so with the TDOA (Time Difference of Arrival) method, only two values, the position on the X axis and Y axis need to be computed to localize an AUV, allowing the minimum number of reference AUVs to be reduced from four to three. It is also assumed that each AUV possesses at least three antennas so that they can receive and decode three arriving messages simultaneously. This is a reasonable design because of the low cost of antennas compared with the whole cost of the AUV. Furthermore, a swarm algorithm like [15] is applied to ensure all AUVs are within one hop, enabling the three reference AUVs to localize all other AUVs.

Suave decides the occasion for re-localization with a dynamic strategy. Whenever the average tracking variance is above a predefined threshold $T_r$, the re-localization procedure begins. Fig. 1 also illustrates the localization procedure in Suave. The selected reference AUVs re-localize themselves by surfacing to receive GPS satellite transmissions. These reference AUVs then re-submerge to localize the other AUVs with TDOA method.

B. Localization of Submerging AUVs

1) Localization Strategies: After re-submerging each reference AUV periodically broadcasts time stamped localization messages containing its current position and sending time. Upon reception of these reference messages, the remaining AUVs localize according to the TDOA scheme. However, to provide constant reference positions after re-submerging, the reference AUVs also need to localize themselves. In order to have constant references, there are two options.
• Option 1: In the first method, each reference AUV remain stationary after surfacing, so that it can broadcast its accurate location to the un-surfacing AUVs for localization. Once the un-surfacing AUVs are out of the communication range of the three reference AUVs, or the localization variances of the un-surfacing AUVs surpass a pre-determined threshold, three new reference AUVs will be selected from the un-surfacing AUVs for the next round of surfacing operation, then serve as a new set of reference AUVs for other AUVs. In this method, during the localization period of the un-surfacing AUVs, all three reference AUVs remain stationary in order to provide accurate location information, and all the other AUVs move towards their individual destinations.

• Option 2: The second method is assisted by an oracle, where an accurate water current velocity is known to each individual reference AUV, so that the reference AUV can accurately calculate its location during the period of submergence.

However, neither of these two approaches is suitable for Suave, because the first slows navigation, and the second is hard to achieve. Thus, we apply a hybrid method where the reference AUVs localize themselves by estimating the velocity of the water current.

A reference AUV maintains its position where it is re-localized in the previous round of localization, represented as \( \vec{P}_t = x_t + j y_t \), where \( j = \sqrt{-1} \), either as a reference AUV or otherwise. After surfacing, its new position is denoted \( \vec{P}_c = x_c + j y_c \). The velocity of the AUV is the combination of the water current and propulsion system velocities between position \( \vec{P}_t \) to \( \vec{P}_c \). Here, we assume that AUVs operate in an open area and that water current velocity does not change significantly between consecutive surfacing periods. Since the propulsion system velocity \( \dot{\vec{v}}(t) = \dot{x}_p(t) + j \dot{y}_p(t) \) is known to the AUV, the average velocity of the water current \( \vec{\mu} = \mu_x + j \mu_y \) can be estimated with:

\[
\int_{t_i}^{t_e} \dot{\vec{v}}(t) dt + \vec{\mu}(t_e - t_i) = \vec{P}_c - \vec{P}_t
\]

After calculating the velocity of the water current, the new position \( P_k \) at time \( k \) will be:

\[
\vec{P}_k = \vec{P}_c + \int_{t_c}^{t_k} \dot{\vec{v}}(t) dt + \vec{\mu}(t_k - t_c)
\]

**Remark 1:** Note that, the water current velocity is estimated by assuming it is constant, which may not be true. The inaccuracy will affect the tracking variance in the IMM, which will in turn affect the surfacing frequency.

2) **Localization with AUV Tracking:** An AUV moves toward the desired destination with its propulsion system. However, the vehicles heading is also passively influenced by currents, which change throughout the mission to produce complicated patterns of movement. The complexity of the movements generated by currents are not easily characterized by a single pattern. Therefore, an IMM estimator [16] is employed to encompass multiple patterns of maneuvering.

The IMM is an adaptive estimation approach that differs from many other methods that assume a particular pattern of movement. The IMM filter accommodates possible movement patterns of the AUV by executing a set of filters in parallel, where each filter corresponds to a single pattern. The overall state is then estimated as a combination of these individual patterns. This paper assumes two AUV movement patterns:

- **uniform motion:** moving along a straight line with a constant velocity, modeled by a Kalman filter (KF).
- **maneuver:** coordinated turn with a constant turn rate and a constant speed, modeled by an extended Kalman filter (EKF).

Fig. 2 shows how these two filters operate in parallel, and the combination of their estimates produces a final estimate of the AUV position. The IMM estimate of AUV location is computed as a weighted sum of the estimates from these multiple mode-matching filters. To identify changes from one pattern of motion to the other, a Markov chain transition matrix characterizing the changes from one motion pattern to another is introduced to update the weighting coefficients. The initial condition of each filter is a combination of the model-conditioned estimates of the two filters in the preceding state.

IV. **TWO KEY COMPONENTS IN AUV LOCALIZATION**

This section introduces the two major components of Suave that improve localization accuracy, reference AUV selection and transmission time adjustment.

Note that, as a prerequisite, both of the following two procedures require the AUVs’ location. In Suave, the estimated position from the IMM for non-reference AUVs and predicted position by computing water current velocity for reference AUVs are applied as the ground truth.

A. **Selection of Surfacing AUVs**

The goal of selecting reference AUVs is to minimize the total localization uncertainty for all AUVs. Suave addresses two issues affecting the accuracy of localization.

The first is tracking variance, which quantifies the differences between its measured and predicted position...
with the IMM. The tracking variance is updated every time when the localization is performed, and maintained by each AUV. The AUVs with the large tracking variance tend to be selected as they are most likely to be inaccurate, and re-localizing these AUVs improves the overall localization accuracy more significantly. In Suave, the AUVs whose tracking variance is larger than the threshold \( T_r \) must be selected and surface to re-localize, even if there are more than three of them, to ensure they will not be lost. However, if one AUV’s tracking variance is smaller than a threshold denoted as \( T_q \), which indicates that it is well tracked, and its position is accurate enough, then this AUV can be the reference AUV immediately without surfacing.

Secondly, AUVs selected at different positions can cause different localization uncertainty. Fig. 3 illustrates an extreme example that may occur in practice (Mathematical derivation is introduced in following paragraph). The uncertainty for each range measurement is assumed to be identical and represented with an ellipse. Localization uncertainty is the uncertainty in the AUV to be localized, be identical and represented with an ellipse. Localization uncertainty is formulated in terms of the current AUVs positions to identify the best combination of reference AUVs.

1) Formulation of the selection problem: Following the TDOA method, the CRLB (Cramer-Rao lower bound) is derived as follows. For notational convenience, the unknown AUV position vector is denoted \( \eta = [x_r \ y_r] \), the reference AUVs position vector \( \xi_n = [x_n \ y_n] \), where \( n \) identifies the index of reference AUV, which is from 1 to 3 in this work. The measurement is denoted as \( \varphi_n := c \Delta t_{n1} \), where \( \Delta t_{n1} \) is the measured time difference of arrival between reference AUV \( n \) and the first reference AUV, and \( \omega_n \) is the measurement noise. The input-output relationship can be expressed as:

\[
\varphi_n = \sqrt{(x_r-x_n)^2+(y_r-y_n)^2+(z_r-z_n)^2} - \sqrt{x_r^2+y_r^2+z_r^2} + \omega_n = h(\eta, \xi_n) + \omega_n
\]  

which can also be stacked into a vector-matrix form:

\[
\begin{bmatrix}
\varphi_2 \\
\varphi_3 \\
\vdots \\
\varphi_N
\end{bmatrix} = \begin{bmatrix}
h(h(\eta, \xi_2)) \\
h(h(\eta, \xi_3)) \\
\vdots \\
h(h(\eta, \xi_N))
\end{bmatrix} + \begin{bmatrix}
\omega_2 \\
\omega_3 \\
\vdots \\
\omega_N
\end{bmatrix} =: \phi
\]  

To solve this nonlinear estimation problem, an iterative least squares estimator may be employed. Based on the estimate \( \hat{\eta}^i \) in the \( i \) th iteration, the estimate in the \( (i+1) \)st iteration is updated as

\[
\hat{\eta}^{i+1} = \hat{\eta}^i + (J^T R^{-1} J)^{-1} J^T R^{-1} [\varphi - \hat{h}(\hat{\eta}^i, \xi)]
\]  

where \( R \) is the covariance matrix of the term \( \omega \), and the Jacobian matrix \( J \) is

\[
J = \frac{\partial h(\eta, \xi)}{\partial \eta} \bigg|_{\eta=\hat{\eta}^i}
\]  

which may be expanded as

\[
J = \begin{bmatrix}
\frac{x_r-x_1}{d_1} & \frac{y_r-y_1}{d_1} & \frac{z_r-z_1}{d_1} \\
\frac{x_r-x_2}{d_2} & \frac{y_r-y_2}{d_2} & \frac{z_r-z_2}{d_2} \\
\frac{x_r-x_3}{d_3} & \frac{y_r-y_3}{d_3} & \frac{z_r-z_3}{d_3} \\
\frac{x_r-x_N}{d_N} & \frac{y_r-y_N}{d_N} & \frac{z_r-z_N}{d_N}
\end{bmatrix}
\]  

with

\[
d_1 = \sqrt{x_r^2+y_r^2+z_r^2}
\]  

\[
d_n = \sqrt{(x_r-x_n)^2+(y_r-y_n)^2+(z_r-z_n)^2}
\]  

After a sufficient number of iterations, the mean square error (MSE) matrix can be obtained as

\[
E[(\hat{\eta}^i - \eta)^T (\hat{\eta}^i - \eta)] = (J^T R^{-1} J)^{-1}
\]  

which may be evaluated based on the estimated \( \hat{\eta} \).

In summary, the AUVs possessing tracking variance greater than \( T_r \) should be selected with highest priority, denoted as \( \alpha \). If \( \alpha \) is greater than 3 then the selection procedure ends. Otherwise, the AUVs whose tracking variance is smaller than \( T_q \) will be selected next, denoted as \( \beta \). If \( \alpha + \beta \) is greater than 3, then the selection procedure ends. Otherwise, with the above derivation, the MSE of localization is computed for any combination of three AUVs with previously selected AUVs included. In this case, there will be \( \binom{\gamma-\beta-\alpha}{3-\alpha-\beta} \) ways to select the reference AUV, where \( \gamma \) denotes the total number of AUVs in the swarm. For each combination, we estimate the MSE of every other AUV and observe the mean. The combination of three AUVs exhibiting the smallest mean is selected as the reference set. This approach requires \( \gamma-\beta-\alpha \times (\gamma-3) \) computations.

B. Transmitting Time Adjustment For Submerging AUVs Localization

Due to long transmission and propagation delays in mobile underwater networks, vehicle movement within this time period can negatively affect localization accuracy, especially when the distance between the reference and target AUV is large or the velocities of the AUVs differ
significantly. Ideally, a target AUV receives all three reference messages simultaneously. However, this is difficult to achieve in practice because reference AUVs prefer to send only one message in each localization procedure to conserve energy. Furthermore, perfect knowledge of vehicle positions cannot be guaranteed. Fig. 4 provides an example to illustrate how limiting the discrepancy between the arrival of the first and last message can mitigate this problem. The Suave algorithm determines the transmission time of the three reference AUVs to maximize the number of vehicles that receive these reference messages within a specified time period \( \delta \). Here, \( \delta \) is a time interval for each AUV which is to be localized, and different AUVs may possess their own time line.

The Suave algorithm tracks each AUV at all times so a rough estimate of their position is always available. Thus, the distance from the three reference AUVs to all vehicles may be denoted according to the following matrix:

\[
\tau = \begin{pmatrix}
\tau_{11} & \tau_{12} & \cdots & \tau_{1N} \\
\tau_{21} & \tau_{22} & \cdots & \tau_{2N} \\
\tau_{31} & \tau_{32} & \cdots & \tau_{3N}
\end{pmatrix}
\]

Let the transmission time of the three reference AUVs be \( T = [t_1, t_2, t_3]\), then the arrival times of these three messages to each AUV are:

\[
\Gamma = \begin{pmatrix}
t_1 + \tau_{11} & t_1 + \tau_{12} & \cdots & t_1 + \tau_{1N} \\
t_2 + \tau_{21} & t_2 + \tau_{22} & \cdots & t_2 + \tau_{2N} \\
t_3 + \tau_{31} & t_3 + \tau_{32} & \cdots & t_3 + \tau_{3N}
\end{pmatrix}
\]

We define:

\[
\theta_i = \begin{cases} 
1 & \text{AUV } i \text{ receives all reference messages in } \delta \\
0 & \text{otherwise}
\end{cases}
\]

Hence, the objective of this optimization problem is to maximize \( \theta \):

\[
\max_T \left( \sum_{i=1}^{N} \theta_i \right).
\]

The transmission time must be positive, requiring the following constraints:

\[
\begin{cases} 
t_1 \geq 0 \\
t_2 \geq 0 \\
t_3 \geq 0
\end{cases}
\]

\[
\theta_i \leq 1 - \frac{\lambda_{mi} - \delta}{M}, m = 1, 2, 3
\]

where \( M \) is a large positive integer. The constraint in Eq.(17) may be explained as follows. If a message is not received within time period \( \delta \), \( \lambda \) is greater than \( \delta \), which implies that \( \frac{\lambda - \delta}{M} \) will be a small positive value because \( M \) is a large positive integer. Thus, \( 1 - \frac{\lambda - \delta}{M} \) is positive, requiring the binary variable \( \theta \) to be 0. Similarly, if all messages are received in the time period \( \delta \), \( \frac{\lambda - \delta}{M} \) will be negative and \( 1 - \frac{\lambda - \delta}{M} \) positive so that the binary variable \( \theta \) can be either 0 or 1. However, \( \theta \) tends to be 1 because of maximization. This enables the optimization problem to be solved in linear time with a standard algorithm such as branch and cut. This optimization problem identifies the transmission time of each reference message that reduces the impact of movement during transmission and propagation, improving the accuracy of localization.

V. ERROR ANALYSIS

The error in AUV localization includes two components,

- location inaccuracy of reference AUVs caused by the estimation error of water current velocity;
- location inaccuracy of the non-surfacing AUVs, which can be attributed to the location error of reference AUVs and the location variance involved in the localization process.

The following analysis examines these two sources of error.

A. Error Analysis of Reference AUVs

In particular, the water current velocity of AUV \( i \) in the \( X \)- and \( Y \)-coordinates is modeled as:

\[
\begin{cases}
\hat{\mu}_{x,i} = \hat{\mu}_{x,i} + n_x \\
\hat{\mu}_{y,i} = \hat{\mu}_{y,i} + n_y
\end{cases}
\]

where \( \hat{\mu}_{x,i} \) and \( \hat{\mu}_{y,i} \) are the estimates of average current velocity in the \( X \)- and \( Y \)-coordinates, and \( n_x \) and \( n_y \) are the respective noise components introduced by variations in current velocity.

Denoting the time duration between consecutive broadcasts from reference AUV as \( \Delta T \), let \( (\hat{x}_{i,i}[k], \hat{y}_{i,i}[k]) \) represent the estimated \( X \)- and \( Y \)-coordinates of AUV \( i \) when broadcasting the \( k \)th message after the \( i \)th surfacing. We thus have

\[
\begin{cases}
\hat{x}_{i,i}[k] = x_{i,i}[0] + \hat{\mu}_{x,i} k \Delta T \\
\hat{y}_{i,i}[k] = y_{i,i}[0] + \hat{\mu}_{y,i} k \Delta T
\end{cases}
\]

where \( x_{i,i}[0], y_{i,i}[0] \) represents the location of AUV \( i \) updated via GPS during the \( i \)th surfacing.

Assuming \( n_x \sim N(0, \sigma_{x_i}^2) \) and \( n_y \sim N(0, \sigma_{y_i}^2) \), the location variance of the reference AUV is

\[
\begin{cases}
\sigma_{x,i,i}^2[k] = \sigma_{x_i}^2 (k \Delta T)^2 \\
\sigma_{y,i,i}^2[k] = \sigma_{y_i}^2 (k \Delta T)^2
\end{cases}
\]
B. Error Analysis of Non-surfacing AUV

Non-surfacing AUVs perform self-localization via the TDOA method [17] with messages received from reference AUVs. The localization error of reference AUVs translates into the measurement noise in (3).

Localization error in the non-surfing AUV can be reduced significantly by exploiting the IMM tracking algorithm introduced in Section III. We next take the Kalman filter, one mode of the IMM estimator, to illustrate the tracking procedure. Define the state vector and two matrices,

\[
\mathbf{L}_{x,l,i}[k] = \begin{bmatrix} x_{l,i}[k] \\ v_{x,l,i}[k] \\ y_{l,i}[k] \\ v_{y,l,i}[k] \end{bmatrix}^T \quad (21)
\]

\[
\mathbf{F} = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)
\]

\[
\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (23)
\]

The state equation for Kalman filtering is

\[
\mathbf{L}_{x,l,i}[k + 1] = \mathbf{F}\mathbf{L}_{x,l,i}[k] + \mathbf{w}_{pr}[k] \quad (24)
\]

where \( \mathbf{w}_{pr}[k] \) is the process noise, which models the uncertainty of an AUV’s mode deviating from the first order kinematic model. The measurement equation can be formulated as

\[
\mathbf{z}_{x,l,i}[k + 1] = \mathbf{H}\mathbf{L}_{x,l,i}[k] + \mathbf{w}_{me}[k + 1] \quad (25)
\]

where \( \mathbf{z}_{x,l,i}[k + 1] \) is a column vector formed by the X- and Y-coordinates of AUV \( i \) based on the TDOA method after the \( (k + 1) \)st broadcast period and \( \mathbf{w}_{me}[k + 1] \) denotes the measurement noise.

The Kalman filtering algorithm can be applied based on (24) and (25). State and measurement equations similar to (24) and (25) can be obtained for the extended Kalman filter [16], but requires an additional dimension to model the rate of coordinate turn in the state vector.

Using the IMM estimator, the state and covariance estimations from the Kalman filter and EKF are weighted and combined to generate the final state estimate \( \mathbf{L}_{x,l,i}[k] \) and covariance estimate \( \mathbf{P}_{x,l,i}[k] \),

\[
\mathbf{L}_{x,l,i}[k] = \mu_{KF}[k]\mathbf{L}^{KF}_{x,l,i}[k] + (1 - \mu_{KF}[k])\mathbf{L}^{EKF}_{x,l,i}[k] \quad (26)
\]

\[
\mathbf{P}_{x,l,i}[k] = \mu_{KF}[k] \left( \mathbf{P}^{KF}_{x,l,i}[k] + \tilde{\mathbf{P}}^{KF}_{x,l,i}[k] \right) + (1 - \mu_{KF}[k]) \left( \mathbf{P}^{EKF}_{x,l,i}[k] + \tilde{\mathbf{P}}^{EKF}_{x,l,i}[k] \right) \quad (27)
\]

where

\[
\tilde{\mathbf{P}}^{KF}_{x,l,i}[k] = (\mathbf{L}^{KF}_{x,l,i}[k] - \hat{\mathbf{L}}^{KF}_{x,l,i}[k])(\mathbf{L}^{KF}_{x,l,i}[k] - \hat{\mathbf{L}}^{KF}_{x,l,i}[k])^T
\]

\[
\tilde{\mathbf{P}}^{EKF}_{x,l,i}[k] = (\mathbf{L}^{EKF}_{x,l,i}[k] - \hat{\mathbf{L}}^{EKF}_{x,l,i}[k])(\mathbf{L}^{EKF}_{x,l,i}[k] - \hat{\mathbf{L}}^{EKF}_{x,l,i}[k])^T.
\]

Derivation of the optimal weighting coefficient \( \mu_{KF}[k] \) is provided in [16].

VI. PERFORMANCE EVALUATION

A. Simulation Settings

The simulations are implemented in Aqua-Sim, an NS-2 based simulator specifically designed for underwater sensor networks [18]. It can effectively simulate acoustic signal attenuation, packet collisions, multipath, fading and so on in underwater sensor networks. In this work, we apply broadcast MAC and static routing in MAC layer and network layer. 10 AUVs are deployed, the transmission range of each AUV is 400 meters which makes it an one-hop network, and the destination is 5000 meters away from where the AUVs are deployed. Without loss of generality, the result is obtained from 1000 runs of execution. The sound speed is constant as 1500 meters/second. During the mission, the re-localization procedure begins whenever the average tracking variance is above a threshold \( T_r \).

The AUVs with tracking variance below \( T_r \) may serve as references without surfacing. A swarm model introduced in [15] is implemented to ensure all AUVs are within one hop of each other. The transmission range is designated as \( R \). A kinematic model [19] is applied to model the water currents as follows,

\[
\begin{align*}
\mu_x &= k_4 + k_1 \lambda_v \sin(k_2 \tau) \cos(k_3 y) + k_1 \lambda \cos(2k_1 \tau) \\
\mu_y &= k_5 - \lambda v \cos(k_2 \tau) \sin(k_3 y)
\end{align*}
\]

where \( \mu_x \) represents the velocity along the x-axis, \( \mu_y \) stands for the velocity along the y-axis. Variables \( k_1, k_2, k_3, \lambda, v \) characterize environmental factors such as the strength of tides and bathymetry. The parameter settings for the simulation are as given in TABLE I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>( \lambda(0.001\pi,0.0001\pi) )</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>( \lambda(0.001\pi,0.0001\pi) )</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>( \lambda(0.02\pi,0.002\pi) )</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( k_5 )</td>
<td>0.0H</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( \lambda(1,0.1) )</td>
</tr>
<tr>
<td>( v )</td>
<td>( \lambda(0.1,0.01) )</td>
</tr>
<tr>
<td>( T_r )</td>
<td>500m</td>
</tr>
<tr>
<td>( T_\theta )</td>
<td>20m</td>
</tr>
<tr>
<td>( T_\nu )</td>
<td>50m</td>
</tr>
<tr>
<td>( \delta )</td>
<td>10m</td>
</tr>
</tbody>
</table>

B. Results and Analysis

1) Effect of water currents: Fig. 5 illustrate a sample path taken by a single AUV. The figure on the left is the ideal path undisturbed by the influence of water currents, while the figure on the right is the actual path incorporating the impact of water currents. This comparison shows that the water current may cause an AUV to navigate where it should not because inaccurate of the location inputs into the navigation system can lead the AUV astray. However, periodic re-localization can ensure that the vehicle eventually reaches the intended location. Thus, with guidance from the Suave algorithm, IMU and other expensive equipment is not necessary for the AUV swarm to arrive at their destinations.

2) Localization accuracy: Fig. 6 shows the accuracy of AUV localization during the entire trip, according to the NMSE (normalized mean square error), computed as \( \text{NMSE} = E[(\Delta r/R)^2] \), where \( \Delta r = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2} \), and \( R \) is the transmission range.
Four different schemes are compared, including Suave, Suave with randomly selected reference AUVs, Suave with no transmission time adjustment, the average of Suave based on the selection of all three reference AUVs combinations, and MLS [20], which perform as a benchmark in our simulation. The results indicate that Suave achieves the best localization accuracy, demonstrating that selecting reference AUV and adjusting the transmission time of reference messages improve the accuracy of localization. MLS achieves localization by means of wide-bandwidth acoustic signals of specific statistical characteristics (maximum length sequence-MLS), but it suffers from mobility and uncertainties from reference AUVs, which reduces its localization accuracy. Fig. 6 also illustrates that increasing the number of surfacing AUV lowers localization error. This improvement occurs because a larger number of surfacing AUV implies fewer non-surfacing AUV need to be localized. It also increases the number of reference messages, significantly improving localization. However, there will be a tradeoff between localization accuracy and cost because surfacing consumes more energy and increases the probability that the swarm is detected.

3) Effect of parameter $\delta$: Fig. 7 shows the impact of parameter $\delta$, the time period in which the number of AUV receiving three reference messages is maximized. The trend suggests that smaller $\delta$ improves localization accuracy. This improvement occurs because a small interval $\delta$ reduces the impact of mobility on localization error, as described in Fig. 4. However, decreasing $\delta$ can only improve localization accuracy so much before the number of AUV that can receive all three reference messages in a very short time interval decreases. Therefore, 10ms was chosen as the default value for $\delta$ in all simulations.

4) Effect of threshold $T_r$: Fig. 8 evaluates the impact of $T_r$, the tracking variance threshold for re-localization. Whenever the average tracking variance of all AUVs reach this value, the selection procedure starts and the reference AUVs surface or start re-localization immediately if surfacing is not required. As $T_r$ increases, localization accuracy decreases because AUVs are not re-localized until the average tracking variance exceeds $T_r$. However, reducing the frequency of surfacing also lowers energy consumption and the probability of detection, creating a tradeoff between localization accuracy and energy consumption and detection.

5) Effect of threshold $T_q$: Fig. 9 illustrates the impact of the threshold on AUVs qualified to serve as reference vehicles directly. This threshold identifies qualified AUV before re-localization based on the tracking variance. AUVs exhibiting low tracking variance of $T_q$ will not need to act as reference vehicles and therefore can avoid surfacing. This can reduce energy consumption, but will impact localization accuracy. Fig. 9 shows that increasing this threshold lowers localization accuracy because lowering $T_q$ makes more AUVs exceed the threshold and serve as reference AUV to localize non-surfacing vehicles.

Increasing $T_q$ may increase the uncertainty in the AUV qualifying to serve as references, and this uncertainty will be passed along to all non-surfacing AUVs seeking to localize, which may increase the overall localization error. The results also show that for fixed $T_q$, smaller $T_r$ achieves...
better localization accuracy because longer missions will cause error accumulation.

6) Effect from number of tracking messages: Fig. 10 examines the impact of the frequency of reference messages on tracking. The results indicate that increasing the number of messages sent within a one second period improves localization accuracy. This agrees with intuition that a larger number of reference messages provides the tracking algorithm with more sample data, offering greater insight into the AUVs motion and improving the accuracy of location estimates. The frequency of reference messages also influences the number of AUV surfacing. Low tracking variance suggests better tracking, which can limit surfacing operations. However, a larger number of reference messages can significantly increase the energy consumed by communications. On the other hand, better localization reduces the energy that must be consumed during surfacing. Due to the lack of information on the cost of surfacing, enhanced models to optimally balance this tradeoff are planned for future research.

VII. RELATED WORK

Despite the significant growth of UWSN research in recent years, techniques for AUV localization remain relatively limited. Carreras [21] proposed a vision-based localization procedure for an underwater robot in a structured environment. This structure consists of a coded pattern placed at the bottom of a water tank and an onboard camera looking downward to provide key features such as absolute and map-based localization through landmark detection and tracking, enabling velocity and three-dimensional position and orientation tracking for the vehicle. While this scheme demonstrates good performance in the experimental setup, it may be difficult to implement in realistic environments because there is no simple way to reproduce a coded pattern on the seabed and the effectiveness of cameras has not been thoroughly tested in large underwater environments.

Maurelli [22] developed a sonar-based localization approach for an AUV in an unknown environment. His approach developed an improved particle filter to represent the vehicle state, which relies on mechanically scanned profiling sonar to acquire range profiles. Experimental results demonstrated the robustness of the approach to noisy measurements. However, this system requires additional equipment to enable the Doppler Velocity Log, significantly increasing the cost of an AUV.

Many acoustic communication-based localization algorithms require extra deployments as references, including a Phero-Trail Location service protocol [23] that projects the path of the mobile sink node onto a two-dimensional plane representing the water’s surface. Mobile underwater nodes are localized by querying sensor nodes on the water’s surface that track the projected path of the mobile underwater node. Ref. [24] localizes vehicles by measuring the time-of-flight to surface buoys. Ref. [25] relies on a set of underwater acoustic transponders. Ref. [26] requires the support of a surface ship and only allows the AUV to travel a short distance from the ship. Clearly, these strategies impose restrictive assumptions regarding the supporting infrastructure that will be available in the environment where the mission is executed, which may not be easily satisfied in many situations.

The particle filter model estimation technique, commonly used to estimate Bayesian models, has been widely applied to AUV technology. Karlsson [27] proposed a particle filter approach for AUV navigation. However, the focus of this study was mapping instead of localization.
Silver [28] presented a new technique for scan matching with sparse, noisy sensors to enable an AUV to estimate its position in a two-dimensional plane. Their approach utilized a particle filter and an approximation of the likelihood of sensor readings based on nearest neighbor distances to approximate the probability distribution over possible poses. Ref. [29] suggested a probabilistic approach for visual cable tracking based on particle filters to identify the appearance and movement of cables in image sequences according to a measurement function.

Ramiro proposed a method to determine the shape of the swarm [30]. It deals with the localization problem of an underwater robotic swarm in the context of the HARNESS project (Human telecontrolled Adaptive Robotic Network of Sensors).

VIII. CONCLUSIONS AND FUTURE WORK

This paper presents Suave, a novel algorithm to localize AUV swarms. Suave consists of two primary steps. The first selects an appropriate set of AUVs for surfaced to update their locations via external sources, such as satellites. The second localizes the non-surfacing AUVs by allowing the surfacing AUVs to serve as reference nodes. In this second step, an IMM filter is employed to improve the localization accuracy, and an investigation that adjusted the transmission times of reference AUVs to address the negative effect from long propagation delay of acoustic waves was presented. Suave proposes an algorithm to select references in an underwater vehicle network to improve the localization accuracy. It also conserves energy by limiting the tracking reference messages and reducing the number of times AUV surface.

In the future, we will further examine energy consumption, including surfacing, communication, and navigation operations. Furthermore, we will extend Suave to multihop networks, so that AUVs can scatter and regroup during a mission.

REFERENCES